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B028415(028)

B. Tech. (Fourth Semester) Examination, Nov.-Dec. 2021

CAICTE ~~(New~~ Scheme)

(ETC Branch)

PROBABILITY THEORY and STOCHASTIC PROCESSES

Time Allowed : Three hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : *Attempt all questions. Every question has four parts. Part (a) is compulsory. Attempt any* ***two*** *parts from (b), (c) and (d).*

Unit-I

1. (a) State Baye theorem. 4

(b) At a certain military installation, six similar radars are placed in operation, it is know that radar's probability of failing to operate before 500 hours of 'on' time has accumulation is 0·06. What are the probability that before 500 hours have elapsed : 8

(i) all will operate

(ii) all will fail

(iii) only one will fail

(c) A student is known to arrive late for a class 40% of the time. If the class meets five each week find

(i) The probability the student is late for at least three classes in a given week.

(ii) The probability the student will not be late at all during a given week. 8

(d) A sub marine attempts to sink an aircraft carrier. It will be successful only if two or ore torpedoes hit the carrier. If the submarine fires three torpedoes and the probability of a hit 0·4 for each torpedo, what is the probability that the carrier will be sunk. 8

Unit-II

2. (a) A honest coin is tossed three times, sketch the application sample space S showing all possible elements. Let X is a random variable that has value representing the number of heads obtained on any triple toss. Sketch the mapping of S onto real line defining X. 4

(b) A random variable X has the distribution function

$F_x(x) = \sum_{n=1}^{12} \frac{n^2}{650} u(x-n)$ find the probabilities : 8

(i) $P[-\infty < X \leq 6 \cdot 5]$

(ii) $P[X > 4]$ &

(iii) $P[6 < X \leq 9]$8

(c) The lifetime of a system expressed in weeks is a Rayleigh random variable X for which 8

$$f_x(x) = \begin{cases} \left(\frac{x}{200}\right) e^{\frac{-x^2}{400}} & , \quad 0 \le x \\ 0 & , \quad x < 0 \end{cases}$$

(i) What is the probability that the system will not last a full week?

(ii) What is the probability the system lifetime will exceed one year?

(d) In a certain junior Olympics javelin throw distances are well approximated by a Gaussian distribution for which $a_x = 30$ & $\sigma_x = 5\,\text{m}$. In a qualifying round, contestants must through further than 26 m to qualify. In the main event the record through is 42 m. 8

(i) What is the probability of being disqualified in the qualifying round?

(ii) In the main event what is the probability the round will be broken.

Unit-III

3. (a) Define the moment's. What do you understand by moment about the origin? 4

(b) A random variable X is uniformly distributed on the interval (–5, 15) another random variable $Y = e^{-\frac{x}{5}}$ is formed. Find $E(Y)$. 8

(c) A random variable X has probability density 8

$$f_x(x) = \begin{cases} \frac{5}{4}\left(1 - x^4\right), 0 < x \le 1 \\ 0 \end{cases}$$

Find :

(i) $E[X]$

(ii) $E[4X + 2]$

(iii) $E\left[X^2\right]$

(d) A random variable X is called Weibull if it has the form $f_x(x) = abx^{b-1}e^{-ax^b}$ where $a > 0$ & $b > 0$ are real numbers. 8

Find :

(i) The mean

(ii) The second moment and

(iii) The variance of x

Unit-IV

4. (a) What do you mean by temporal characteristics of random process? 4

(b) Given the Random variable

$$X(t) = A\cos(w_0 t) + B\sin(w_0 t)$$

where w_0 is a constant and A and B are uncorrelated zero mean random variable having different density function but the same variance σ^2, show that $X(t)$ is wide-sense stationary but not strictly stationary. 8

(c) For a stationary eragodic process with no periodic components, the auto correlation function is

$$R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$$

Find the mean value and variance of the process $X(t)$. 8

(d) A small store has two check-out lanes that develop waiting lines if more than two customers arrive in any one minute interval. Assume that a Poisson process describe the number of customers that arrives for check-out. Find the probability of a waiting line if the average rate of customer arrivals is : 8

(i) 2 per minutes

(ii) 1 per minute

(iii) ½ per minute

Unit-V

5. (a) What is white noise? 4

(b) Assume a random process has power spectrum

$$S_{xx}(w) = \begin{cases} 4 - \left(\frac{w^2}{9}\right) & , \quad |w| \le 6 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Find :

(i) the average power

(ii) r.m.s. bandwidth 8

(c) What are properties of cross-power density spectrum? 8

(d) Determine the cross correlation function corresponding to cross power density spectrum. 8

$$S_{xy}(w) = \frac{6}{(9 + w^2)(3 + jw)^2}$$